Final Report for 2016 USGS NEHRP Award G16AP00030,
Beyond the Corner Frequency: Connecting Dynamic Source Models to Seismic Observations

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Abstract
For over 40 years seismologists have been studying small and moderate sized earthquakes using a relatively simple mapping between 'easily' observed characterizations of finite-source effects on seismic waveforms and two early dynamic models of circular crack ruptures done by Brune [1970] and Madariaga [1976]. We propose to expand this connection between the underlying dynamic models of earthquake rupture and easily observed descriptions of finite-source effects. Recently Kaneko and Shearer [2014, 2015] revisited the Madariaga model and re-emphasized the strong azimuthal variations in the far-field pulse shape due to finite source effects even for relatively simple crack models with no material heterogeneity or irregular rupture geometries. In this project, we have tried to connect the azimuthal variations of these models to the features that are relatively easy to measure and quantify for moderate earthquakes. The overall goal is to be able to determine the underlying dynamical properties for large suites of moderate (M2-5) earthquakes. For instance, do certain fault segments produce super-shear ruptures more readily due to their geometry and geology? Answering this requires the ability to study large populations of ruptures on a well known fault. In 2016, we focused on three main tasks: 1) Publishing our software package for determining the finite source properties of moderate earthquakes using the second moments technique, 2) examining the recording geometries required to resolve the finite source properties related to rupture velocity, and 3) exploring how the underlying features of the dynamic models of Kaneko and Shearer are reflected in the second moments measurement and inversion schemes. Perhaps our most interesting result to date is the identification of a combination of measurements that can reveal super-shear ruptures even for crack-like models that inherently have only moderate directivity.

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The overall goal of this project is to connect the dynamic models of earthquake ruptures done in Kaneko and Shearer [2014, 2015] with the finite source estimation scheme developed in McGuire [2004]. In the long term, we think this combination will allow us to determine more insight into the dynamics of large populations of moderate earthquakes by analyzing real datasets in ways that would be expected to illuminate the key features of rupture dynamics based on simulations. The data analysis scheme relies on the ability to estimate the second degree moment tensor of an earthquake's rupture [Backus, 1977a,b] from the spatial variations in the far-field pulse shape [Silver 1983, Doornbos 1982]. Our progress in 2016 on this project came in three primary areas: 1) Publication of a Matlab toolbox for doing the second moment's estimation, 2) evaluation of the second moments inverse problem, 3) direct comparisons of 2nd moments quantities to dynamical models of rupture. Below we detail each of these.

1) Publication of the Matlab Toolbox: Following the comments of the NEHRP panel specifically requesting that we do this, we prioritized the proposed task of creating a relatively easily used public version of the 2nd moments inversion code. This software was published in 2017 in the *Electronic Seismologist* section of *Seismological Research Letters* [McGuire, 2017] and is available from both the SRL website and on McGuire's website at WHOI. To make the convex optimization scheme relatively simple, this version uses Matlab's Robust Control Toolbox. This is a great solution for the U.S. academic community in terms of ease of use. Future efforts will work to extend this to a completely free version in Python/ObsPy and/or R. However, there is currently less standardization in the convex optimization routines available in those languages compared to Matlab.

2) Evaluation of the Second Moments Inverse Problem via dynamic models: While we have done evaluations of the second moments inversion scheme for some ad-hoc statistical slip distributions and a number of real earthquakes [McGuire, 2004; Chen and McGuire 2016], we have not previously done the inversion on a realistic dynamical rupture model with a known rupture velocity and fracture energy. The models of Kaneko and Shearer give us a chance to validate the resolution of the inversion scheme. Figure 1 shows three models of circular ruptures. From a static stress drop point of view they are nearly identical in that their rupture areas, and slip distributions are nearly identical. However, owing to the variations in rupture velocity and epicenter location, the variations in their moment-rate functions are significant. This is reflected in the second moment quantity termed the characteristic duration, tau_c, which varies from 0.13 to 0.21 s for the three models. Because of this variation in rupture duration (e.g. 1/corner frequency), if one blindly applied the Brune model to these three ruptures, the result would be three very different stress drop estimates with about an order of magnitude variation despite the fact that their rupture areas and slip distributions are nearly identical.

Figure 2 shows simulated inversion results for synthetic datasets calculated from the far-field P and S-wave pulses produced by the models in Figure 1. Each simulated dataset contains a number of P and S waves chosen at random over the focal sphere. The estimated second moment quantities are shown as a function of the # of measurements in the simulated dataset. It takes about 10 measurements to accurately estimate the rupture length of all 3 models, which is about 550 m. Compared to using the Brune model, where the three estimates of circular radius would vary by about a factor of 2, the results in Figure 2 show that the second moment approach can recover the identical rupture length's of these three models within about 5%. It is also able to properly determine that the upper two models are perfectly symmetrical bilateral ruptures (|V0|=0) while identifying that the lower left model is unilateral propagation from east to west (V0= [1.6 km/s, 0 km/s]). We are using these models to determine what level of station coverage will be required with future instrument deployments aimed at resolving super shear ruptures.

3) Connection of second moment observations to rupture dynamics.

Fundamentally, if we want to resolve the rupture dynamics of moderate ruptures in real datasets, we need clearly measurable quantities that are based on the far-field radiation and that are highly diagnostic of the underlying rupture features that we are interested in. We are currently working on identifying the most robust way to go after super-shear ruptures. There are a few quantities that are commonly estimated from far-field P and S-waves. Most noteably, the corner frequency, *fc*, of a Brune spectrum fit is routinely measured with spectral ratios. For good datasets, it is often also possible to simultaneously estimate both fc and the spectrum fall-off exponent *n* (rather than constraining it to be a Brune-like 2.0) at each station. In our second moments inversion scheme [McGuire, 2004], we use the related time-domain quantities, the variances of the moment-rate functions because they allow us to enforce positivity on the Empirical Green's Function deconvolution (unlike the spectral-fitting approach). These variances are a measure of duration and we usually present them as twice the square-root of the variance, which is termed the characteristic duration *tau_c*, which is a function of the slowness of the phase at the source. Hence it is different for P and S waves at the same station. Thus, we have at least six quantities that are routinely measurable and could be automated for large datasets, they are *fc, n, and tau_c* for both the P and S wave at each station.

All 3 of these quantities vary rapidly over the focal sphere, particularly for pulse-like unilateral ruptures with high rupture velocities. Figure 3 shows some examples of the variation of fc and tau_c, see Kaneko and Shearer [2014, 2015] for more examples. For any earthquake the individual values of these quantities are not particularly diagnostic of the underlying rupture dynamics because to first order they are simply measures of earthquake duration. The higher-order information that they contain is most obvious in various ratios between the P and S-wave of each quantity. Figure 4 shows an example of the power of these quantities at identifying super-shear rupture velocities for the 'Asyymetrical Ellipse' ruptures of Kaneko and Shearer [2015]. Observable measurements are shown for 5 different models. All of the models are unilateral ruptures with approximately the same rupture length, but they vary strongly in rupture velocity. The observations are shown color-coded by the rupture velocity of the underlying dynamic model. There is a clear separation of super-shear ruptures from the sub-shear ones. This is particularly impressive as the Kaneko and Shearer models are still crack-like ruptures due to their underlying friction law. A friction law that resulted in pulse-like ruptures would produce an even stronger variations. We are still exploring the parameter space and working with Yoshi Kaneko on his catalog of available models. However, Figure 4. indicates that there are likely systematic measurement schemes that can identify or rule-out super-shear ruptures routinely.

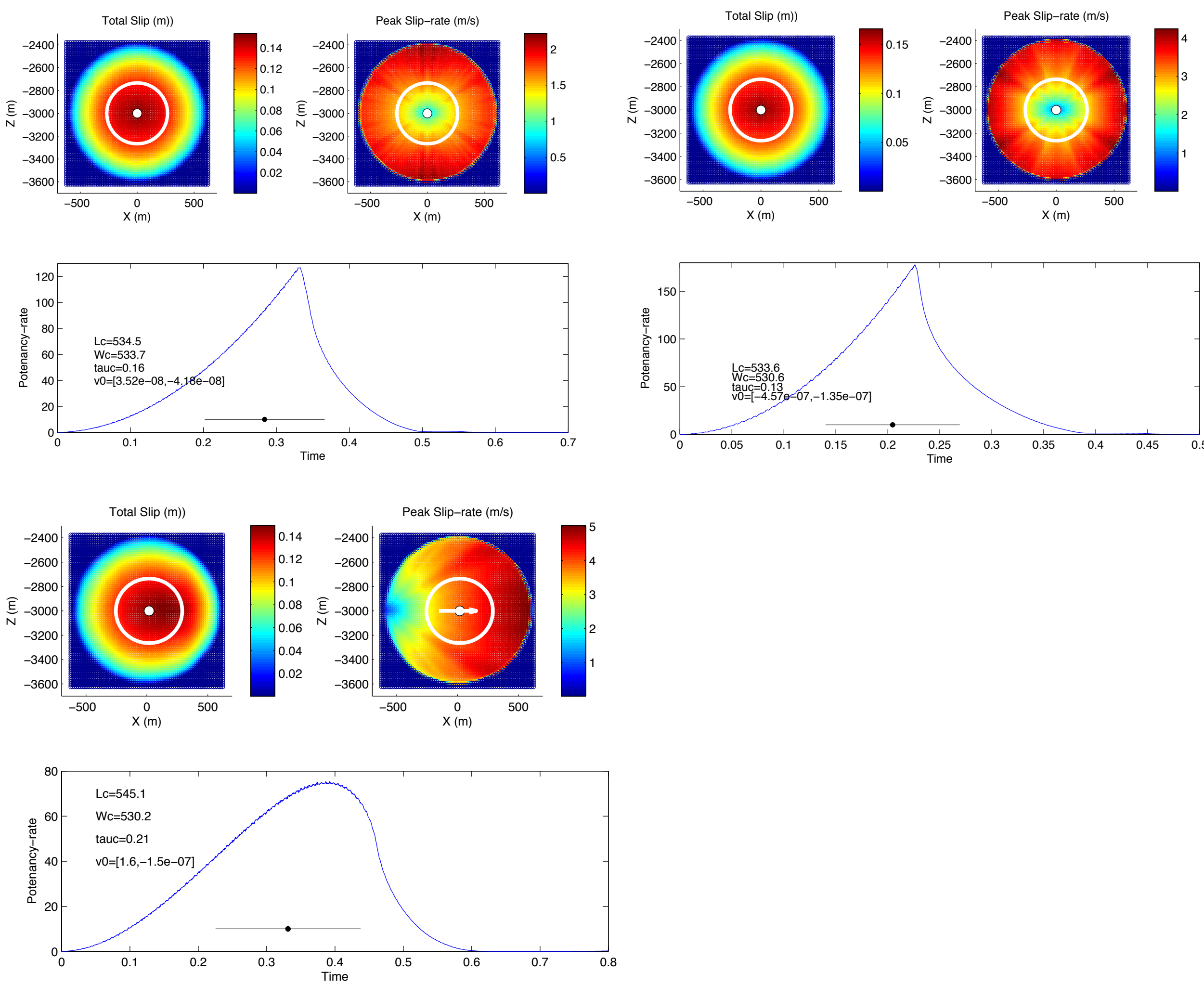

Figure 1. The second moments of 3 different circular rupture models of Kaneko and Shearer. These are essentially a recreation of the Madariaga model with some slight variations. The upper left panels show the Slip and Slip-rate distributions and moment-rate function for a circular rupture that initiates at its center and propagates outwards with a rupture velocity that is 60% of the shear-wave speed. The upper-right panels show a similar model but with Vr= 90% of shear wave speed. The lower left panels show a similar rupture but this time the epicenter is at the lefthand edge of the circular patch and the rupture propagates relatively unilaterally with a velocity of 0.9 times the shear-wave speed. Each moment-rate function is labelled with the second moment quantities that describe the Length (Lc), Width (Wc), duration (tau_c), and rupture propagation velocity (v0).

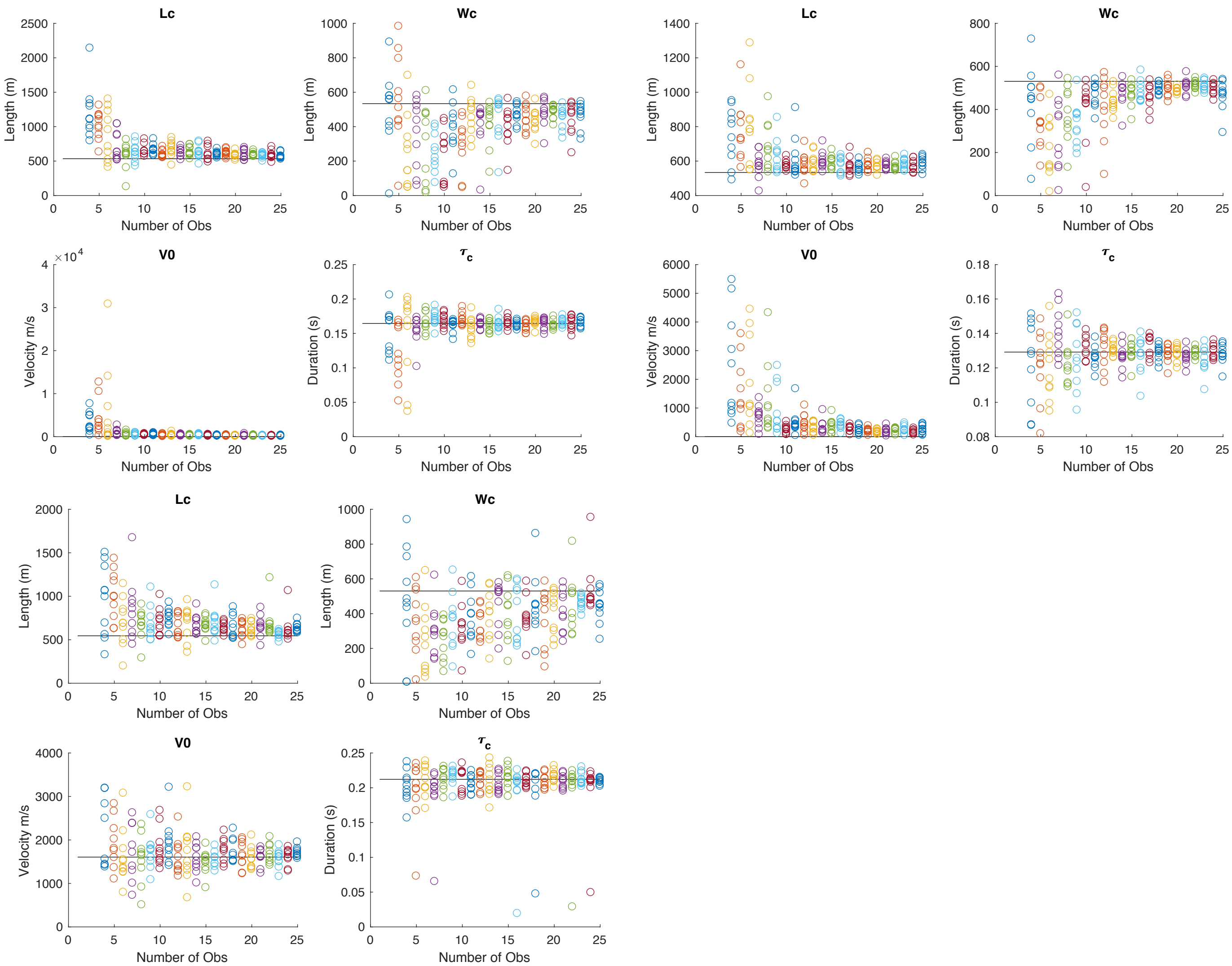

Figure 2. Results of inversions of simulated data for the 3 circular crack models in Figure 1 in the same configuration. For each model, synthetic datasets were created by randomly choosing a particular number of observations around the focal sphere. The estimated values of Lc Wc, tau_c, and |V0| are plotted as a function of the number of observations used in the simulated dataset. The true values of the quantities are given by the horizontal lines. Note |V0| =0 for the top two models, but about 1.7 km/s for the lower left (unilateral) model. Each circle denotes the inversion from one simulated dataset. In general, the second moments inversion requires about 10 measurements to properly resolve the rupture length.

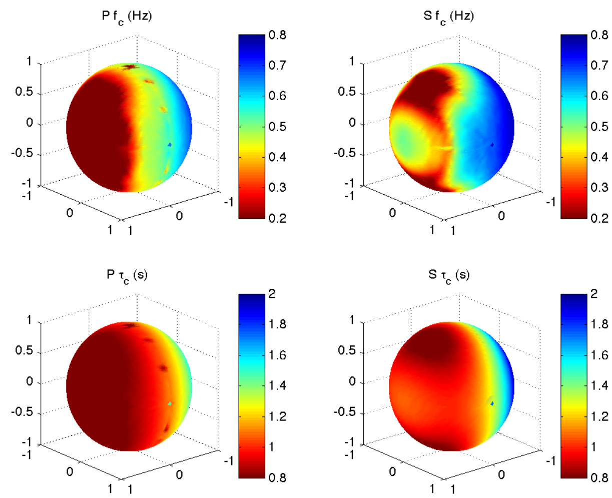

Figure 3. Examples of the spatial variations in P and S-wave corner frequencies (Hz) and Tau_c (seconds) values for the AsymmEllip1.6 super-shear model of Kaneko and Shearer 2015.

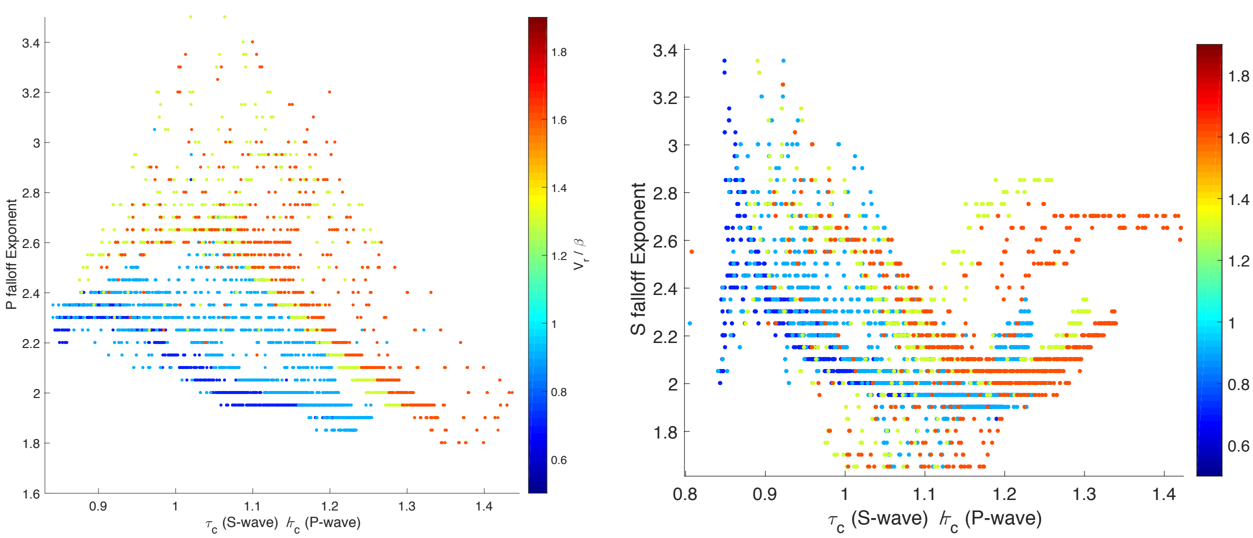

Figure 4. Measurements of fall-off exponent versus tau_c (S)/ tau_c (P) for the P-wave (left) and S-wave (right) for 5 different rupture models of Kaneko and Shearer including the one in Figure 3. Each station around the focal sphere that recorded each model is plotted as a unique point color-coded by the rupture velocity of the underlying dynamic model. While each individual quantity that we can measure varies significantly, there are particular ratios that allow the super-shear ruptures to be separated from the sub-shear ruptures.

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